MATHEMATICAL MODELLING OF A CONTINUOUS SETTLING APPARATUS

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Initiated by the need to produce quartz-free bentonite as a filler substance for machine greases, the mathematical model of a continuously-operating settling apparatus has been elaborated, together with a computer programme to evaluate experimental data and compare them with the theoretical ones. Though the experimental flow pattern deviates to various extents from the ideal laminar flow used in the theoretical approach, the granulometric curves measured and computed on the product agree fairly well. The method described can be used for modelling and optimization of continuous settlers.

Bentonite is one of the most frequently occurring clay minerals; it is widely used in both its original and its modified, organophilic form in many fields of industry (food, textile, pharmaceutical, varnish and paint industries). Its role in oil production is gaining in importance, particularly in deep drilling.

Our task was to investigate the experimental possibilities of producing quartz-free bentonite as a filler for the production of machine greases by settling.

The experiments were carried out in a continuously-operating settling apparatus. The settling was desired to produce a fraction with radius under 2 μm which, as has been shown in separate experiments, is free of quartz. For the evaluation of the experimental data and the mathematical modelling of the apparatus a simplified model of the settling process was considered.

Principle of calculation

Let it be supposed that a liquid containing solid particles is flowing laminarly as a layer of thickness L on top of a static bulk liquid, as shown in Fig. 1. Within the layer the linear rate of flow decreases from its maximum value $v(L) \equiv v_{max}$ at L to $v(0) \equiv 0$ at $L=0$; accordingly, the path of a solid particle entering the continuous settler at ($h=0$, $l=L$) will be a parabola as long as the settling takes place in the moving medium, and a straight line, normal to the direction of flow, in the static bulk. If the overflow is taken only from the moving layer, a residence time $t^*(r)$ can be defined for every particle with radius r by

$$t^*(r) \equiv \frac{L}{v_{sed}(r)} \tag{1}$$

where v_{sed} is the rate of sedimentation, characterized by the following property: if the settling time t as determined by the applied input flow rate and the size of the settling apparatus is equal to or greater than $t^*(r)$, *i.e.* $t \geqq t^*(r)$, then the particle will settle; otherwise it leaves the settler in the overflowing liquid.

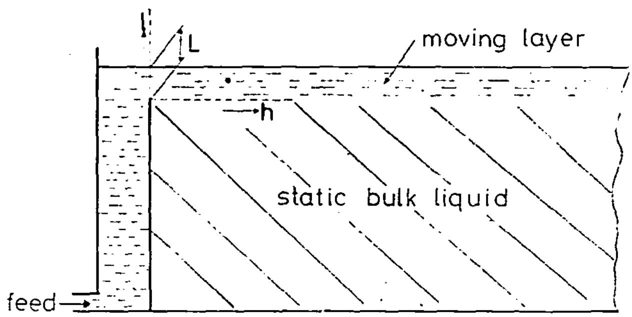

Fig. 1. Sketch of the settling apparatus

Now, in possession of the density function of the particle size distribution, $f(r)$, for the original suspension and assuming ideal functioning of the settler, it would be a very simple task to find the particle size distribution of the suspension in the overflowing liquid and that remaining in the settler as well. It can easily be shown that, taking the mean residence time $\bar{t}$ of the flow

$$\bar{t} = \frac{H \cdot L \cdot a}{w} \tag{2}$$

where H is the length,
a is the width of the settler and
w is the volumetric flow rate,

a sedimentation rate $\bar{v}_{sed}(r)$ and hence a particle radius $\bar{r}$ may be obtained *via* equ. (1). Particles having radii equal to or greater than $\bar{r}$ will settle, while the others will leave the settler. In non-interacting suspensions the rate of sedimentation $v_{sed}(r)$ is described best by Stokes' law [1].

Unfortunately, the density function of the residence time of flow in real settling apparatus, $E(t)$, deviates to various extents from that valid for ideal laminar conditions because the actual flow pattern is never free of small backward currents:

$$E(t) \neq E(t)_{id} \equiv \frac{2t_0^2}{t^3} \quad (t \geqq t_0)$$

where $t_0 \equiv \dfrac{L}{v_{max}}$ is the time necessary to reach the end of the settler with the maximum flow rate. Additionally, the input and output of the suspension correspond only approximately to the conditions set at the beginning. To overcome this difficulty the distribution of the residence time should be determined empirically. However, this semiempirical approach becomes so complicated that actual calculations have to be carried out using a computer.

Experiments and calculations

The experiments and calculations were performed according to the scheme shown in Fig. 2.

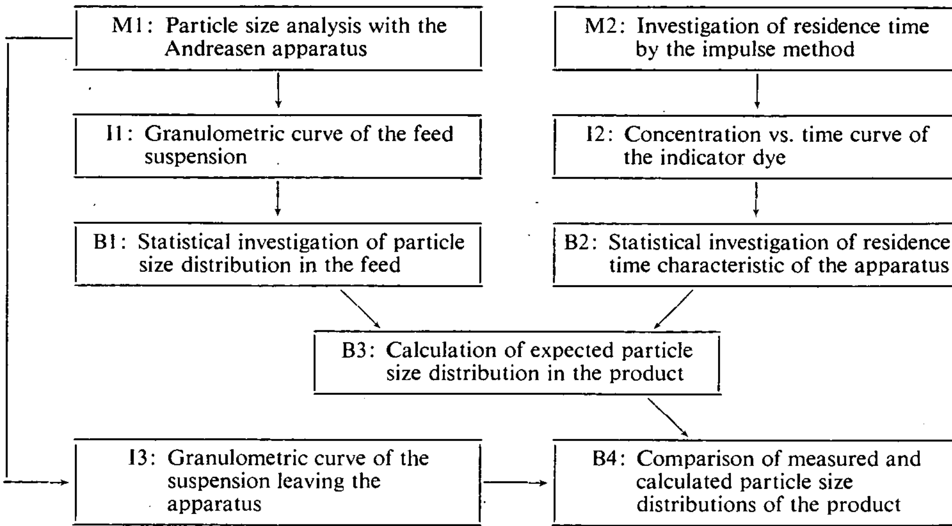

Fig. 2. Flow sheet

The flow-sheet contains the segments of the computer programme (B1, B2, B3, B4), the blocks of the input data (I1, I2, I3) and the symbolic blocks of the experimental methods (M1, M2).

As may be seen from the Figure, the *experimental work* consisted of two steps: *analysis of the particle size distribution* of the suspension fed into and leaving the apparatus, and the fluid mechanical description of the apparatus.

The granulometric curves $(Y_g(r))$ of the suspensions (I1, I3) were measured by means of Andreasen's static settling apparatus (M1) [2]. In block B1 the particle size distribution is calculated from the granulometric curves by the equation:

$$Y(r) = 1 - Y_g(r).$$

The density function of the particle size distribution according to mass, $f(r)$, is determined by numerical derivation, applying the Douglass—Avakian method [3]. (The Douglass—Avakian procedure fits a fourth-degree polynomial to seven points of a curve by the method of least squares and takes the derivative analytically.)

The *fluid mechanical study* of the apparatus comprised the other part of the experimental work. By measuring the residence time of the fluid flow, it was possible to determine empirically how closely the flow pattern in the apparatus approaches ideal laminar flow. For this purpose the impulse method was the most suitable (M2). Briefly, an aqueous solution of an appropriate dye (in our case erioglaucin A) is injected into the liquid at the place of input (the amount was M g), and the concentration of the dye, $c(t)$, is measured in the overflowing fluid (I2). In segment B2 of the computer programme the calculation of the density function of the residence time distribution, $E(t)$, is carried out according to the formula:

$$E(t) \equiv \frac{w}{M} c(t).$$

The residence time distribution function, $G(t)$, is calculated by numerical integration according to the trapezoidal rule [3]:

$$G(t) = \int_{t_0}^{t} E(\tau)\, d\tau.$$

The mean residence time, $\bar{t}$, can also be calculated by numerical integration, applying Simpson's rule [3]:

$$\bar{t} = \int_{t_0}^{\infty} \tau E(\tau)\, d\tau.$$

By definition the average thickness of flow is calculated from equ. (2) using $\bar{t}$:

$$L = \frac{\bar{t} \cdot w}{H \cdot a}.$$

The time $t^*(r)$ necessary for particles with a radius between r and $r+dr$ to settle with a sedimentation rate v_{sed} in the moving layer can be calculated by means of Stokes' law:

$$t^* = \frac{9L\eta}{2r^2(\varrho_s - \varrho_f)\, g}$$

where ϱ_s is the density of the suspended solid particles,
ϱ_f is the density of the dispersion medium,
g is the gravitational constant,
η is the coefficient of internal friction of the medium.

In block B3, which is of key importance in the computer programme, the *expected* particle size distribution in the suspension leaving the apparatus is calculated.

The particles remaining in the flowing layer for time t, which is less than that needed to pass the distance L (*i.e.* $t < t^*$), will get into the fine fraction (product), their mass ratio* in it being $G(t^*) \cdot f(r)\, dr$. To obtain the total mass ratio in the overflow, this function has to be integrated numerically in some interval $(0, r_{max})$:

$$Q \equiv \int_{0}^{r_{max}} G(t^*(r)) \cdot f(r)\, dr.$$

The particles having an actual residence time $t \geqq t^*$ will leave the moving layer and settle in the static bulk liquid filling part of the settling channel. To get as good a value as possible for $G(t^*(r))$ at every point t^*, $G(t^*(r))$ is calculated most suitably by quadratic interpolation.

* The mass of suspension in unit mass of feed, both on a water-free basis.

The density function of the particle size distribution in the suspension leaving the settler, $\varphi(r)$, is determined by the relative mass of the fraction with particle sizes between r and $r+dr$:

$$\varphi(r) \equiv \frac{G(t^*(r))f(r)\,dr}{Q}.$$

An analogous calculation can be used for the determination of the particle size distribution for the suspension remaining in the apparatus.

The numerical integration of $\varphi(r)$ (*e.g.* using the trapezoidal rule) provides the particle size distribution function of the product:

$$\Phi(r) = \int_0^r \varphi(\tau)\,d\tau.$$

The granulometric curve is computed from:

$$\psi_g(r) \equiv 1-\Phi(r).$$

In the last segments of the computer programme, B4 and I3, the measured and calculated particle size distributions and the granulometric curves of the product are compared.

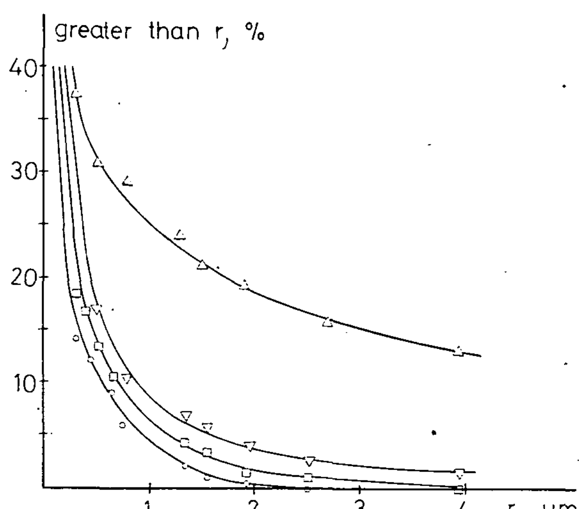

Fig. 3. Granulometric curves
of the suspension fed into and leaving the settler
—△—△—: suspension fed in
suspensions settled with various volumetric flow rates:
—▽—▽—: w=4.66·10⁻⁶ m³ s⁻¹
—□—□—: w=3.16·10⁻⁶ m³ s⁻¹
—○—○—: w=1.08·10⁻⁶ m³ s⁻¹

Experimental results and their discussion

The experimental results are listed in Table I. It is seen from the accumulations in the settler at various flow rates that optimum separation could be achieved at the lowest input flow rate. The quality of fractionation is illustrated far more impressively in Fig. 3, where the granulometric curves of the product obtained at various flow rates are shown. In this fraction no quartz could be detected by *X*-ray or optical methods.

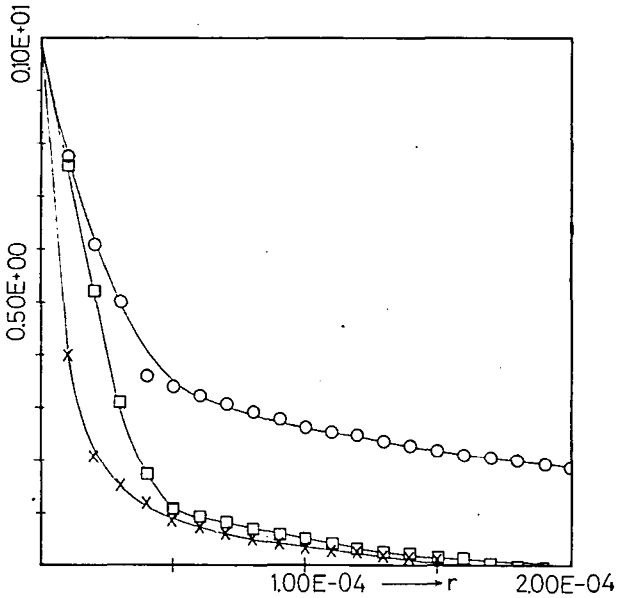

Fig. 4. Granulometric curves (results of the computerized approximation)
$w = 1.08 \cdot 10^{-6}$ m^3 s^{-1}
—○—○— : suspension fed in
—□—□— : calculated
—×—×— : measured

Figure 4 shows the results of the computerized approximation. The measured and calculated granulometric curves do not entirely coincide. This can be explained by the fact that the thickness of flow, L, as calculated from the mean residence time, $\bar{t}$, is an average value. In the calculation of the settling times t^* pertaining to the radii r of individual particles, *this* value was taken as the sedimentation height. As mentioned earlier, this is an idealized picture from which the true flow pattern in the apparatus shows considerable deviations. Nevertheless, the computerized approach is still statisfactory and can be used for the description of the settling process in a given apparatus.

The principle of calculation and the computer programme outlined above can be used for modelling similar settling problems, in the planning of settling apparatus, and for the optimization of the operational parameters of existing apparatus.

Table I

Experimental data

Volumetric flow rate [10^{-6} m^3 s^{-1}]	1.08	3.16	4.66
Mean residence time [s]	402.3	353.1	187.9
Mean thickness of flow [10^{-2}·m]	0.218	0.555	0.384
Reynolds number	42.34	116.2	175.0
Solid content of the suspension in the feed [10^{-4} kg m^{-3}]	1.006	1.006	0.926
Solid content of the suspension leaving the apparatus [10^{-4} kg m^{-3}]	0.766	0.826	0.805
Accumulation [10^{-4} kg m^{-3}]	0.240	0.180	0.121
Mass ratio leaving the apparatus [%]	76.1	82.1	86.9

References

[1] *Wolfram E.:* Kolloidika II/1. Egységes jegyzet (Colloid Chemistry II/1. University notes) Tankönyvkiadó Budapest 1969.

[2] *Batel, W.:* Einführung in die Korngrössenmesstechnik, Springer-Verlag Berlin 1971.

[3] *Sherwood, T. K., Reed, C. E.:* Applied Mathematics in Chemical Engineering, McGraw-Hill Book Co., Inc. New York 1939.

МАТЕМАТИЧЕСКОЕ МОДЕЛИРОВАНИЕ ОСАДИТЕЛЬНОГО ОБОРУДОВАНИЯ НЕПРЕРЫВНОГО ДЕЙСТВИЯ

Ш. Катона и П. Фейеш

Цель работы заключалась в получении суспензии бентонита без кварца, которая может использоваться в качестве наполнителя смазки. Разработана математическая модель непрерывного осадительного оборудования. Оценка результатов измерений, а так же их сравнение с рассчитанными данными были проведены на ЭВМ. Хотя экспериментально определенный характер потока различается более или менее от теоретически предпологаемого ламинарьного потока, измеренные и рассчитанные гранулометрические составы продукта довольно сходны. Описанный метод может использоваться для моделирования и оптимализациия непрерывних осадительных оборудований.